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Analytical Study of A Two-Phase Model For Steady Flow of Blood in A Circular Tube

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Abstract:
The present paper deals with a mathematical model of blood flow through narrow circular tube. The model consists of a core region of suspension of all the erythrocytes assumed to be a power law fluid and a peripheral cell-depleted layer of plasma as a Newtonian fluid. The system of differential equations has been solved analytically. The expressions for velocity profile, Bluntness parameter, flow rate, the ratio of core hematocrit to discharge hematocrit (Hc/H_D), apparent viscosity (μapp), and the ratio of tube hematocrit to discharge hematocrit (H_T/H_D) and shear stress at the wall have obtained. Some of them have been discussed through graphs.

Key words: Two-phase blood flow, cell-depleted layer, Bluntness, Hematocrit, apparent viscosity.

I. Introduction:

Blood is composed of two major components; the cellular component and the plasma component. In an average adult, the blood volume is approximately 5 litre of which approximately 55% to 60% is plasma and the remaining portion is cellular. More than 99% of the cellular component is composed of red blood cells. The most common way to quantify the percent of blood that is cellular is by quantifying the packed red blood cell volume, which is termed the hematocrit. Hematocrit or red blood cell (RBC) concentration and the shear rate are the principal independent variables for describing the apparent viscosity of blood and other RBC suspension. The formation of RBC aggregates at low shear rates may affect blood flow in the microcirculation. The experimental evidence suggests that hematocrit distribution in the microvasculature is not uniform: RBCs tend to concentrate near the center of the vessel, thus forming and RBC-depleted plasma layer near the wall. RBCs are non-uniformly distributed not only within, but also among the micro vessels. The heterogeneous distribution of RBCs and other blood cells has important implications for microvascular hemodynamic and molecular transport.

The two important mechanisms that cause non-proportional distribution of RBCs and plasma in the microcirculation are "cell screening and "plasma skimming". The cell screening mechanism **(Cokelet,s 1976 [1]; Pries *et al.*, 1981 [2])**, involves direct cell-cell and cell wall-fluid mechanical interactions near the orifice of a side branch.

These interactions cause the RBC trajectories to deviate from the fluid stream lines; which would exist in the absence of the cells. The plasma skimming mechanism is related to the non uniform distribution of RBCs at the inlet cross- section of arteriolar bifurcations, in particular the formation of a cell-depleted layer near the vascular wall **(Tateishi *et al.*, 1994 [3]; Yamaguchi et. al, 1992 [4]).** For the "ideal" plasma skimming case, when the flow fraction in the branch is less than 0.5 the discharge hematocrit in the branch becomes lower than in the parent vessel.

When blood flows through tubes, the two-phase nature of blood as a suspension becomes important as the diameter of the red blood cell (RBC) becomes comparable to the tube diameter. The following are some of the effects observed in vitro and in vivo:

(i) Fahraeus - Lindqvist effect: dependence of apparent viscosity on tube diameter;
(ii) Fahraeus effect: dependence of tube or vessel hematocrit on tube diameter;
(iii) Existence of a cell-free or cell-depleted layer near the wall;
(iv) Blunt velocity profile;
(v) Phase separation effect: disproportionate distribution of red blood cells and plasma at vessel bifurcation.

Fournier [5] have been developed several models to interpret these effects. **Pries *et al.* [6]** reviewed biophysical aspects of micro-vascular blood flow in vivo as well as in vitro.

Nair et. al. [7] used a two-phase model for the blood in modeling transport of oxygen in arterioles. They considered a cell-rich cone surrounded by a cell-free plasma layer.

In the cell-rich core, the radial hematocrit distribution was expressed as a power law profile with maximum at the center of the tube. The thickness of the cell-free layer was chosen on the basis of geometrical consideration in terms of RBC size and radius of the tube. However, the

dependence of the thickness on the cell free layer on hematocrit was not taken into account. **Seshadri and Jaffrin [8]** modeled the outer layer as cell-depleted having a lower hematocrit than in the core. The apparent viscosity and the mean tube hematocrit were taken from the measurements obtained in glass tubes. The concentration of RBCs in the cell-depleted layer was assumed to be 50% of that in the core. **Gupta *et. al.*, [9]** divided the outer layer into a cell-free plasma layer and cell-depleted layer. In both these studies, the velocity profile in the core was assumed to follow a power law. **Pries *et al.* [10,11,12]** derived empirical relationship of the relative apparent viscosity and mean tube hematocrit as parametric functions of tube diameter and discharge hematocrit from in vitro **Pries *et al.* [10,12]** and in vivo **Pries *et al.* [11]** data.

Numerical modeling can provide information for various hematocrits. Hematocrit is known to affect the viscous properties of blood **(Merril, E.W. [13] and Chien, S. et. al. [14])** and physiological abnormalities in hematocrit are associated with diseases which alter the blood composition (**Chien, S. et. al. [14]; Halvorsen, S.[15]; Skovborg, R. [16] and Leblond, P.F. et. al. [17])** . For example, over production of red blood cells (policythemia) increases whole blood viscosity, while iron deficiency (anemia) decreases blood viscosity. Changes in blood composition may influence wall shear stress patterns in the arterial system, which may in turn play a role in the sequence of arterial diseases. Effect of hematocrit on wall shear rate in oscillatory flow has been studied by **Kathleen and John [18]** and found that increase in hematocrit produced a decrease in the peak wall shear rate in both the straight and curved artery models and a corresponding decrease in wall shear rate reversal on the inside wall of the curved artery model.

Das *et al.*, [19] considered the effect of nonaxisymmetric hematocrit distribution on non-Newtonian blood flow in small tubes. Eccentric hematocrit distribution is considered such that the axis of the cylindrical core region of red cell suspension is parallel to the axis of the blood vessel but not coincident. Human blood is described by Quemade's rheological model and cat blood is described by Casson's model. Velocity distribution, shear stress, apparent viscosity and Fahraeus effect have been calculated numerically. These are strongly influenced by the eccentricity factor, the core radius and the tube hematocrit. **Maithili Sharan and Popel [20]** proposed a two-phase model for flow of blood in narrow tubes with increased effective viscosity near the wall. The model consists of a central core of suspended erythrocytes and a cell-free layer surrounding the core. A system of nonlinear equation is solved numerically to estimate bluntness, core radius and core hematocrit. Variation of apparent viscosity and tube hematocrit with the tube diameter and the discharge hematocrit in vitro have been discussed. **Davod Alizadehard *et al.*, [21]** investigated the deformation of RBCs in micro vessels for a variety of vessel diameter (8-50μm), Hematocrit (20-45%) and shear rates (20-150S^{-1}) and comparing the apparent viscosity with experimental results.

The aim of the present investigation is to study the flow of blood as a two-phase model. The behavior of blood is considered as power law in core region and cell-depleted layer as Newtonian fluid. Analytical expressions for velocity profile, bluntness parameter, flow rate, ratio of core hematocrit to discharge hematocrit (H_C/H_D), apparent viscosity and ratio of tube hematocrit to discharge hematocrit (H_T/H_D), shear stress at the wall have obtained. The results are discussed graphically.

II. Mathematical Analysis:

The geometry of the model is shown in Fig.1. The steady laminar two layer model for the blood flow within a cylindrical tube of radius R consisting a central core of radius rh and effective viscosity μc which contains an erythrocyte suspension of uniform hematocrit Hc and a cell-free layer outside the core containg plasma with an effective viscosity μo. The blood is considered as non-Newtonian power law fluid in core region and plasma is Newtonian fluid in cell free layer.

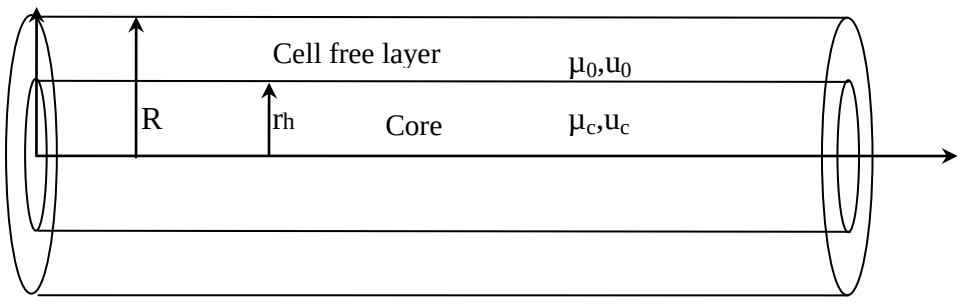

Fig.1. Geometry of the flow model

2.1 Governing equation and boundary conditions:-

The constitutive equation of motion for incompressible steady fully developed flow in a tube reduces to:

$$-\frac{\partial p}{\partial z}+\frac{\mu_c}{r}\frac{\partial}{\partial r}\left[r\left(\frac{\partial u_c}{\partial r}\right)^n\right]=0, \quad 0\le r<r_h \tag{1}$$

for the central core with red blood cells and

$$-\frac{\partial p}{\partial z}+\frac{\mu_O}{r}\frac{\partial}{\partial r}r\left(\frac{\partial u_0}{\partial r}\right)=0, \quad r_h<r<r \tag{2}$$

for the cell-free layer, where u_C and u_0 are the velocities in the cone and plasma layer respectively, p is the hydraulic pressure and r and z represent the radial and axial direction in the tube.

The boundary conditions are:

(a) the velocity gradient varnishes along the axis of the tube:

$$\frac{\partial u_c}{or}=0 \qquad at\ r=0 \tag{3}$$

(b) No slip condition is assumed at the wall;

$$u_0=0 \quad \text{at } r=R \tag{4}$$

(c) The velocity and shear stress are continuous at the interface of plasma and the core:

$$\text{(i)} \qquad u_c=u_0 \quad \text{at } r=r_h \tag{5}$$

$$\text{(ii)} \qquad \mu_c\frac{\partial u_c}{\partial r}=\mu_0\frac{\partial u_0}{\partial r} \text{ at } r=r_h \tag{6}$$

2.2 Solution of the problem

The solution of equation (1) and (2), subject to the boundary conditions (3) - (6) is given by

$$u_c(\xi)=-\frac{R^2}{4\mu_0}\frac{\partial p}{\partial z}\left[1-\lambda^2+\frac{\mu_0}{\mu_c}\alpha\lambda^{\frac{n+1}{n}}\right]\left[1-\frac{\frac{\mu_0}{\mu_C}\alpha}{1-\lambda^2+\frac{\mu_0}{\mu_c}\alpha\lambda^{\frac{n+1}{n}}}\xi^{\frac{n+1}{n}}\right];\ 0\le\xi\le\lambda, \tag{7}$$

$$u_0(\xi)=-\frac{R^2}{4\mu_0}\frac{\partial p}{\partial z}\left[1-\xi^2\right];\ \lambda\le\xi\le1 \tag{8}$$

where

$$\xi=\frac{r}{R},\ \lambda=\frac{r_h}{R},\ \alpha=2^{\frac{2n-1}{n}}\left(\frac{n}{n+1}\right)\left(\frac{R\frac{\partial p}{\partial z}}{\mu_C}\right)^{\frac{1-n}{n}}$$

α=1 for n = 1 (Newtonian fluid) and $\frac{\partial p}{\partial z}$ is pressure gradient along the length of the tube.

Velocity in the core $u_c(\xi)$ can be expressed as

$$u_c(\xi)=u_{max}\left(1-B\xi^{\frac{n+1}{n}}\right) \tag{9}$$

where,

$$u_{max} = -\frac{R^2}{4\mu_0}\frac{\partial p}{\partial z}\left[1-\lambda^2+\frac{\mu_0}{\mu_c}\alpha\lambda^{\frac{n+1}{n}}\right] \quad (10)$$

$$B = \frac{\frac{\mu_0}{\mu_c}\alpha}{1-\lambda^2+\frac{\mu_0}{\mu_c}\alpha\lambda^{\frac{n+1}{n}}} \quad (11)$$

The parameter B is the bluntness of the velocity profile. When n = 1 and $\mu_c \to \mu_0$ *then* $B \to 1$` which give the velocity profile becomes parabolic throughout the entire cross-section of the tube and fluid in both layer in Newtonian.

The volumetric flow rate of the blood is given by

$$Q = 2\pi R^2\int_0^{\lambda} u_c(\xi)\,\xi\,d\xi + 2\pi R^2\int_{\lambda}^{1} u_0(\xi)\,\xi\,d\xi \quad (12)$$

The expression for the flow rate Q is obtained as the evaluation of integrals in (12) with the velocity equations (7) and (8) as:

$$Q = -\frac{\pi R^4}{8\mu_0}\frac{\partial p}{\partial z}\left[1-\lambda^4+\frac{2(n+1)}{3n+1}\left(\frac{\mu_0}{\mu_c}\alpha\right)\lambda^{\frac{3n+1}{n}}\right] \quad (13)$$

Mass balance of the cells in the tube is defined as:

$$Q\,H_D = 2\pi\pi^2\int_0^1 \xi\,u(\xi)\,h(\xi)\,d\xi \quad (14)$$

Where H_D is the discharge hematocrit and h (ξ) is hematocrit function related to core hematocrit H_C as:

$$h(\xi) = \begin{Bmatrix} H_c & 0 \le \xi < \lambda \\ 0 & \lambda < \xi \le 1 \end{Bmatrix} \quad (15)$$

Using (7) and (8) in (14) with (15) we obtain

$$QH_D = -\frac{\pi R^4 H_c}{8\mu_0}\frac{\partial p}{\partial z}\left[2\lambda^2(1-\lambda^2)+\frac{2(n+1)}{3n+1}\frac{\mu_0}{\mu_c}\alpha\,\lambda^{\frac{3n+1}{n}}\right] \quad (16)$$

The ratio H_C/H_D can be obtain from equations (13) and 16) as:

$$\frac{H_C}{H_D} = \frac{1-\lambda^4+\frac{2(n+1)}{3n+1}\frac{\mu_0}{\mu_C}\alpha\lambda^{\frac{3n+1}{n}}}{2\lambda^2\,(1-\lambda^2)+\frac{2\,(n+1)}{3\,n+1}\frac{\mu_0}{\mu_C}\alpha\lambda^{\frac{3n+1}{n}}} \quad (17)$$

Equation (13) can be written as:

$$Q = -\frac{\pi R^4}{8\,\mu_{app}}\frac{\partial p}{\partial z} \quad (18)$$

Where μ_{app} is the apparent viscosity of total tube flow given by

$$\mu_{app} = \frac{\mu_0}{\left[1-\lambda^4+\frac{2\,(n+1)}{3\,n+1}\frac{\mu_0}{\mu_c}\alpha\lambda^{\frac{3n+1}{n}}\right]} \quad (19)$$

The tube hematocrit H_T is defined as:

$$H_T = 2\int_0^1 h(\xi)\,\xi\, d\xi \qquad (20)$$

Using equation (15) in (20), we get

$$H_T = \lambda^2 H_C \qquad (21)$$

The average velocity of the blood $\left(\bar{U}\right)$ is given as

$$\bar{U} = \frac{Q}{\pi R^2} \qquad (22)$$

From equation (10) and (22) the maximum velocity U_{max} can be expressed as:

$$\frac{u_{max}}{\bar{U}} = \frac{2\mu_{app}}{\mu_0}\left[1-\lambda^2+\frac{\mu_0}{\mu_C}\alpha\lambda^{\frac{n+1}{n}}\right] \qquad (23)$$

The shear stress at the wall is defined as

$$\tau_w = -\frac{1}{2}\frac{\partial p}{\partial z}R \qquad (24)$$

Using equation (18), and (22) in (24), the expression for the shear stress at the wall is obtained as

$$\tau_w = \frac{4\,\mu_{app}\,\bar{U}}{R} \qquad (25)$$

Equation (17), (19) and (21) express H_C, μ_{app} and H_T in terms of λ, H_D, μ_0 and μ_C.

III. Results and Discussion:-

In order to discuss the results of the theoretical model proposed in the study, the analytical expression for velocity profile, Bluntness parameter, flow rate, the ratio of core hematocrit to discharge hematocrit (H_C/H_D), apparent viscosity, the tube hematocrit to discharge hematocrit (H_T/H_D) and shear stress at the wall have been obtained. It may be noted that if we put n=1 in present model the results are obtained for both layer is Newtonian.

To discuss the problem, the Bluntness, B; ratio H_c/H_D,apparent viscosity, μapp and ratio H_T/H_D obtained analytically in equation (11), (17), (19) and (21) respectively have been plotted in Figures 2 to 8. For numerical calculations we take $R = 100\ \mu m,\ \frac{\partial p}{\partial z} = -75x10^3\ dyne/cm^3,\ \mu_C = 3.8\ cP,\ \mu_0 = 1.2cP.$

The parameters B in equation (11) is the bluntness of the velocity profile in core. The parameter depends on the thickness of the cell-free layer. Figure 2 show the variation of bluntness parameter with tub radius R for n = 3/4 and n = 5/4. It is observe that the numerical values of B for n=5/4 are less than that for n = 3/4. Bluntness parameter B is plotted in Figure 3 with λ for different values of non-Newtonian parameter. For n = 3/4 bluntness parameter B first decreases upto λ= 0.2 and then increases upto λ = 0.6 and again decreases upto λ = 1.

Bluntness parameters profile is near about similar for n = 1, and n= 5/4 but the values for n = 1 in greater than that of n =5/4.

Figures 4 and 5 show are variation of ratio H_C/H_D with λ and with R for different values of n. H_C/H_D decreases with λ fastly upto λ= 0.4 and then decreases slowly for n = 1 and n = 5/4 but increase upto λ=0.5 then decrease fastly upto λ = 0.6 and again increase. From figure 4 it is observe that when n < 1 the character in very different. Figure 5 shown that H_C/H_D increases fastly upto R= 125μm and then slow effect is obtained for n = 3/4 whereas H_C/H_D decreases very slowly for n = 5/4. Numerical values for n= 5/4 of H_C/H_D are greater than that for n = 3/4.

The variation of apparent viscosity (μapp) with λ for different value of non-Newtonian parameter n is shown in figure 6. Mapp increase slowly with λ upto 0.6 and fastly for n = 1 and n = 5/4 but the character is very different for n = 3/4. The trend of figure are same for n = 1 and n = 5/4 but numerical values for n = 5/4 for different λ are greater than that of n = 1.

From figure 7, it is observed that the ratio H_T / H_D increase with λ for n = 1 and n = 5/4 in similar trend but increases fastly upto λ= 0.4 then decreases upto λ= 0.7 and again increases.

Effect of tube radius R on H_T / H_D is plotted in Figure 8 for n = 3/4, 5/4. H_T / H_D increase with R for n = 3/4 and decreases with R for n = 5/4. Numerical values for n = 5/4 of H_T / H_D are greater than that for n = 5/4.

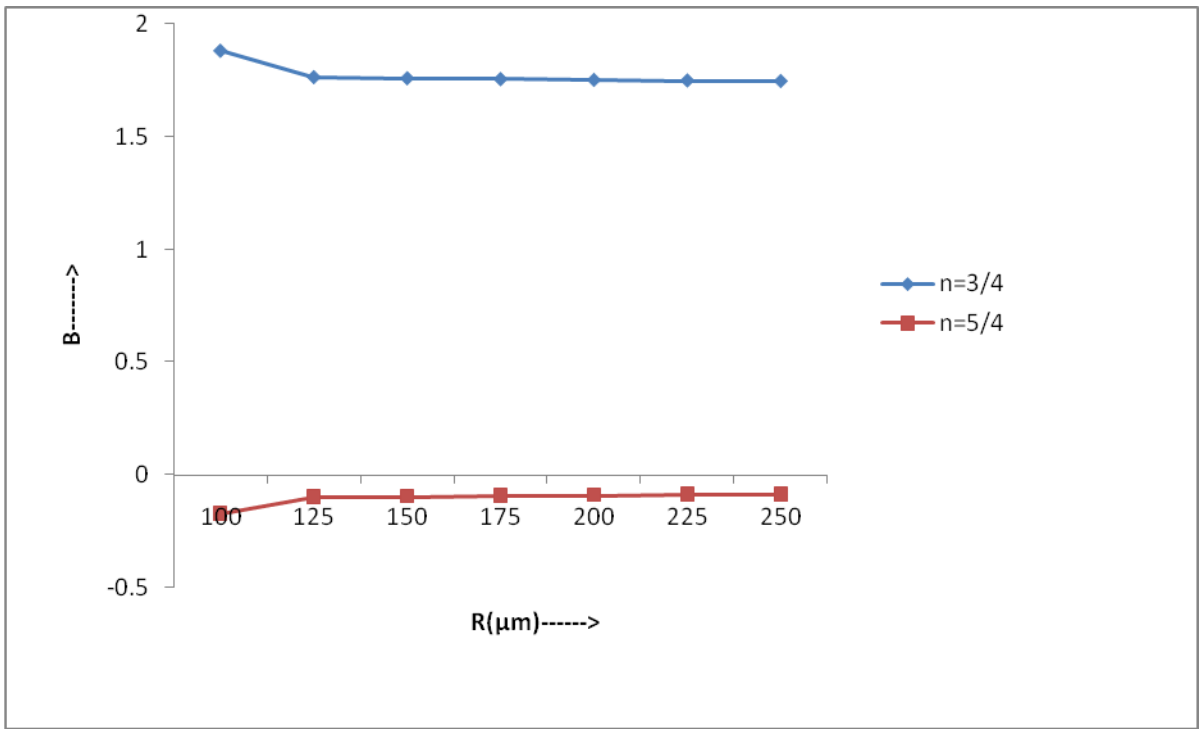

Fig. 2: Variation of Bluntness parameter (B) with R

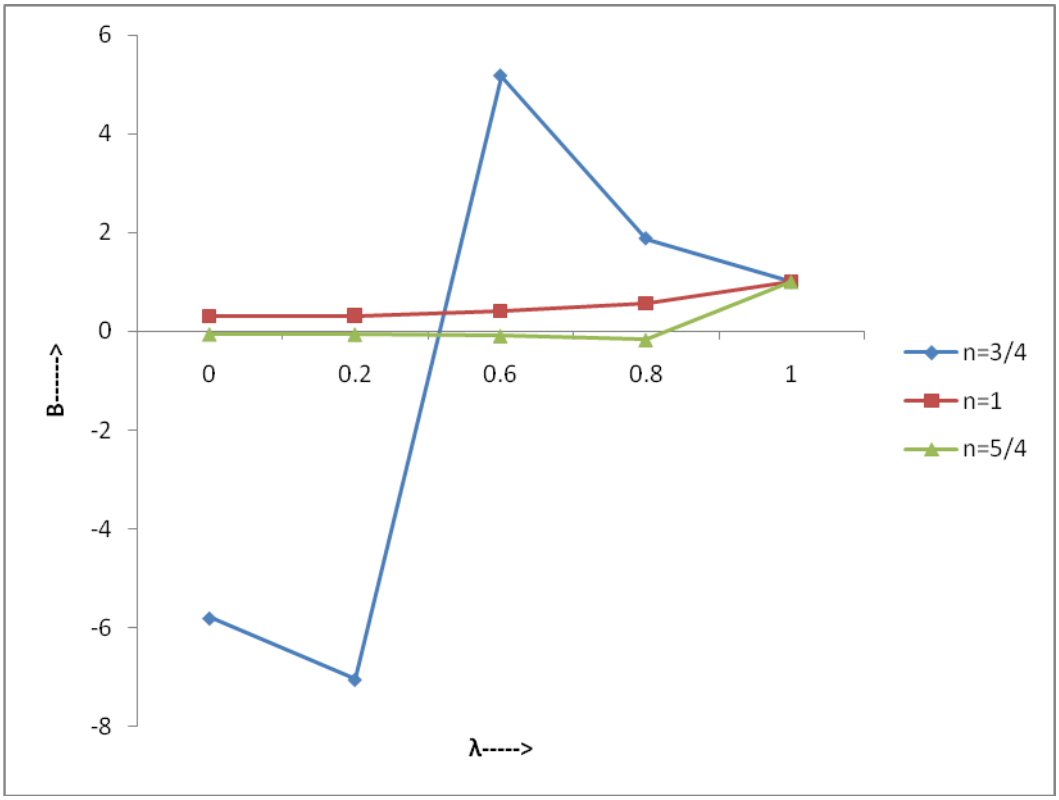

Fig.3: Variation of Bluntness parameter (B) with λ

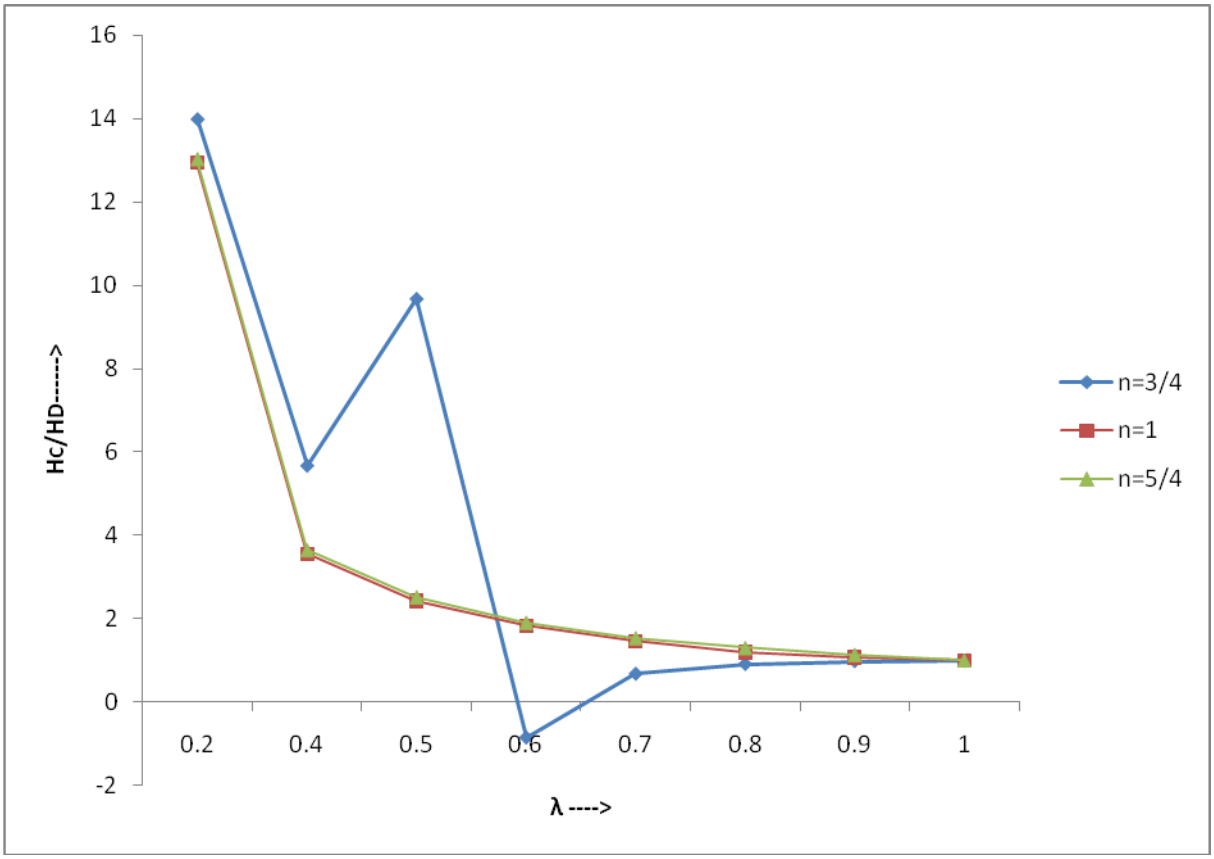

Fig.4: Variation of HC/HD with λ

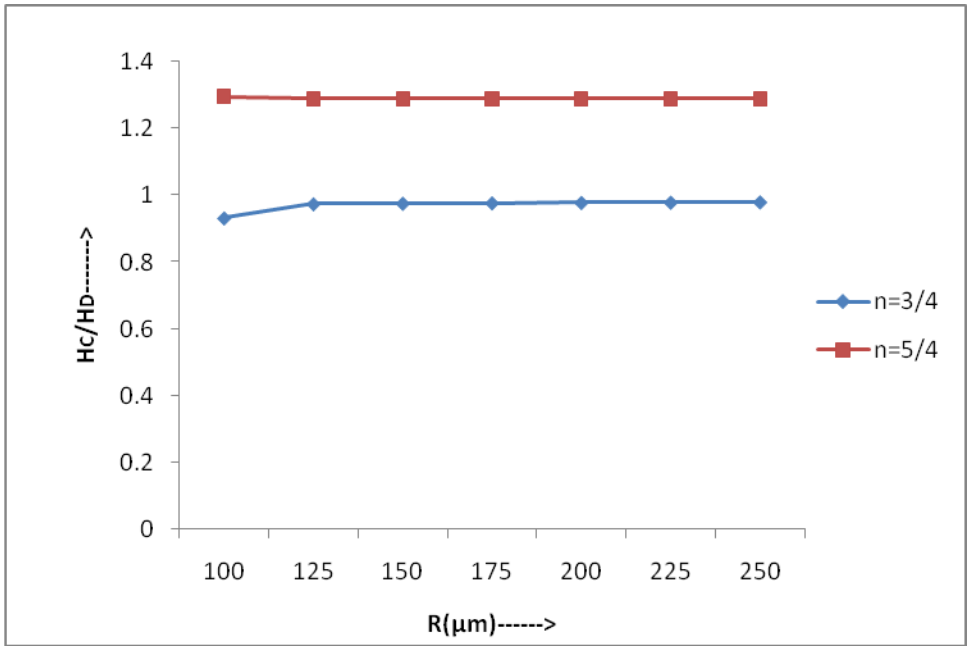

Fig. 5: Variation of HC/HD with R

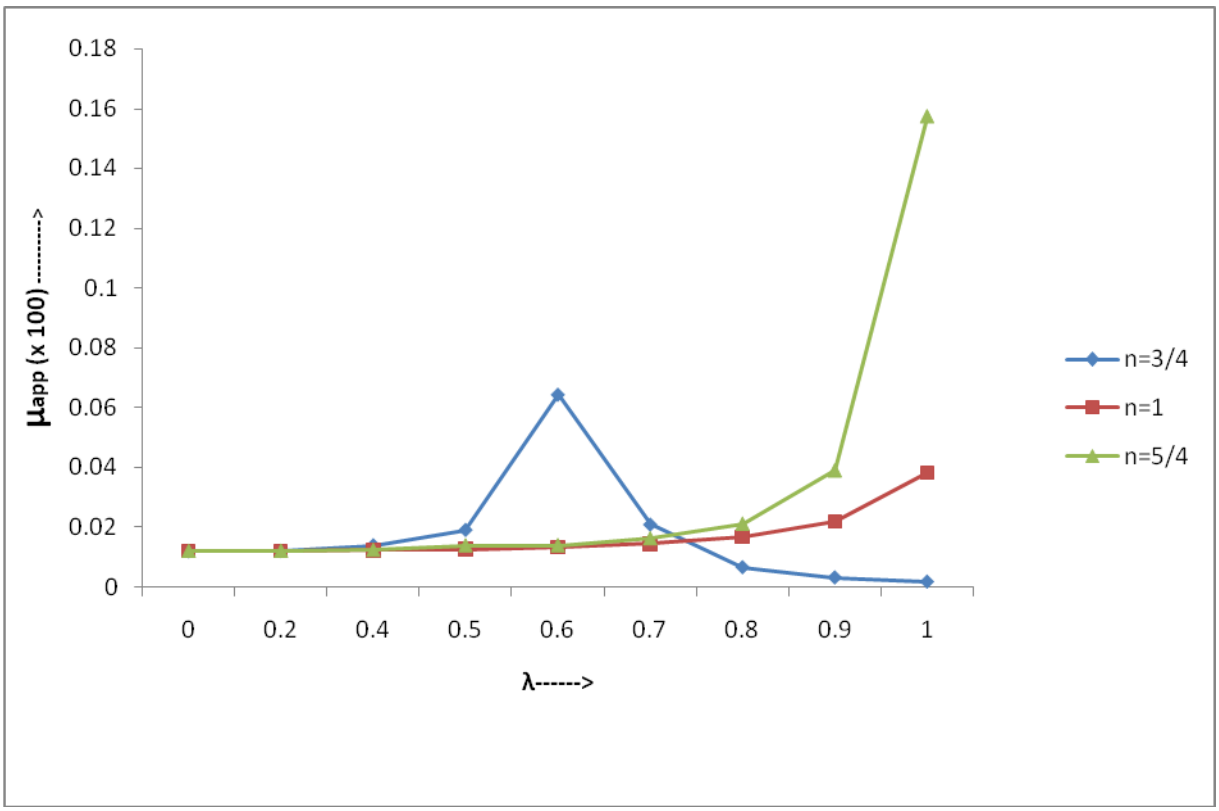

Fig. 6: Variation of apparent viscosity (μapp) with λ

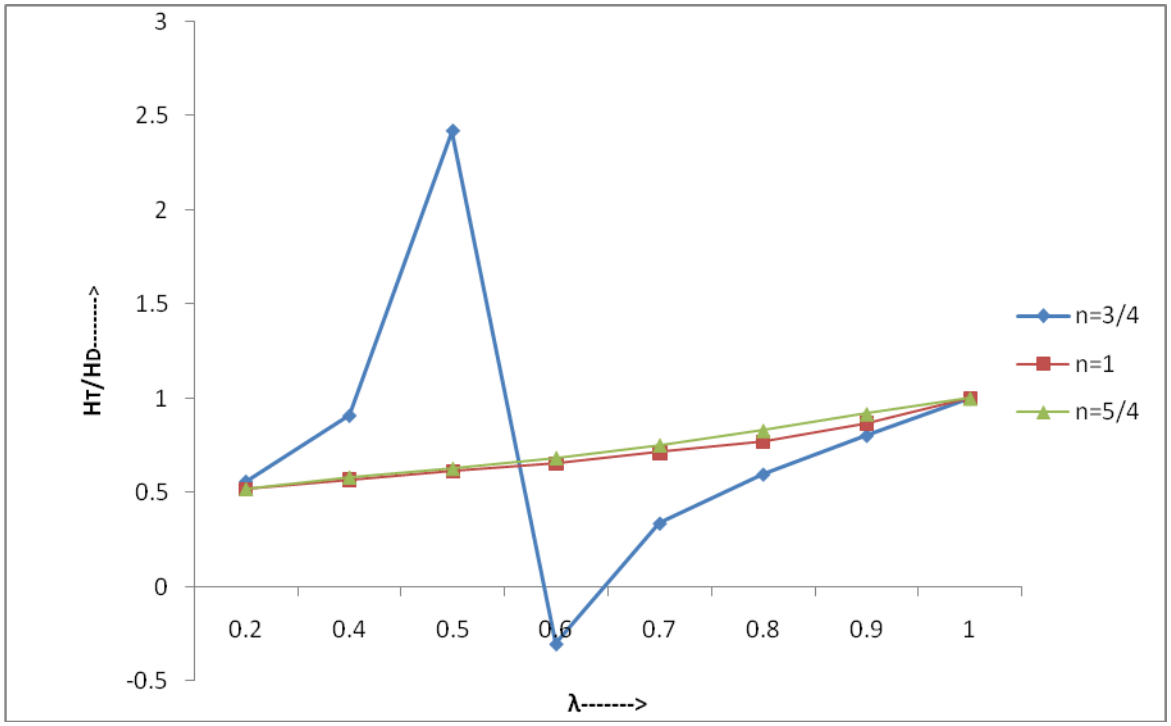

Fig.7: Variation of HT/HD with λ

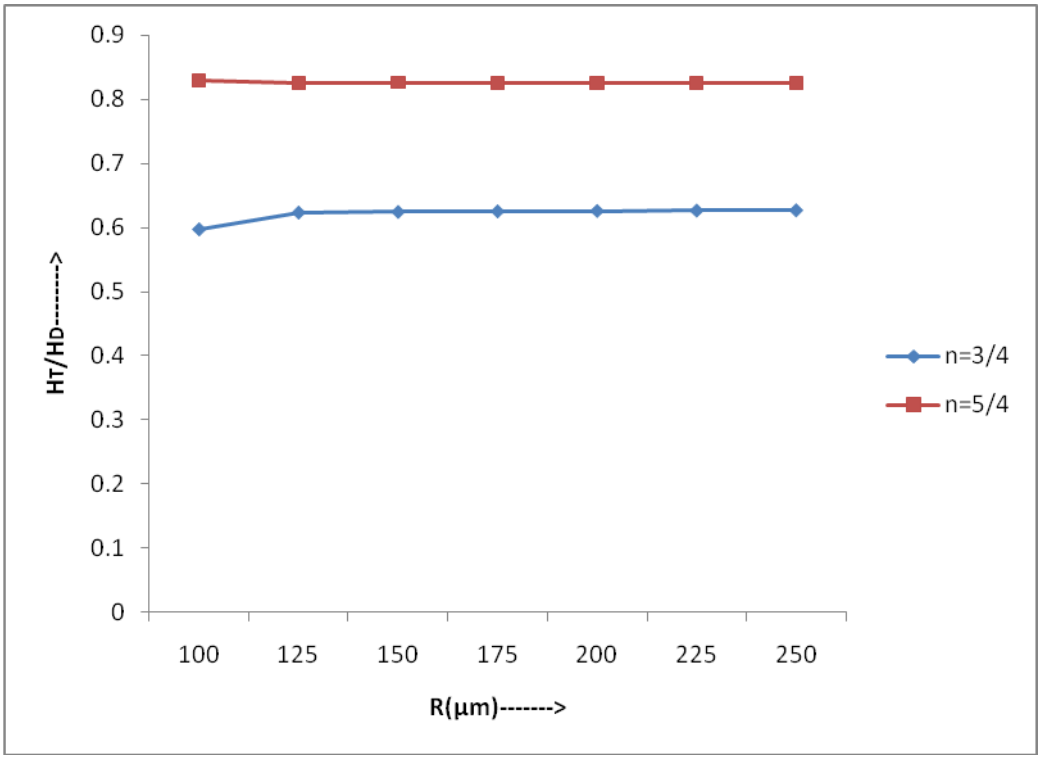

Fig. 8: Variation of HT/HD with R

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